Use of metal detectors for electromagnetic induction local 3D imaging

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Abstract

A recently launched BMBF-funded German R&D on mine sensor technologies for humanitarian demining is focussed, in its first phase, on sounding of the potential of mathematical methodology of end-of-pipe data analysis of conventional metal detectors, with the aim of significant reduction of false alarm rates. Two R&D lines, each of them complemented by forward modelling of the metal detection process, will be followed in parallel: (1) advanced signal analysis, employing modern tools (e.g. neural networks, wavelets) for features extraction and classification, and (2) local EMI imaging of 3D subsurface patterns, employing tomographic methods, with numerical solution of ill-posed inverse problems and their regularization by exhaustion of a-priori knowledge (e.g. signatures of mines and/or soils). The co-ordinated, interdisciplinary approach is sketched, with a focus on the 3D imaging line.

1. Introduction

In humanitarian demining, the classical metal detector (MD) is still a working horse in post-conflict search for buried mines [1]. The operational principle of the MD is electromagnetic induction (EMI): Scanning above the sensed subsurface volumes, the MD generates time-dependent magnetic fields and senses anomalies in the electromagnetic response from the sensed volume. The time-dependent primary magnetic fields induce eddy currents in metallic objects, and the eddy currents generate secondary magnetic fields. Usually, the primary magnetic field is generated by one coil, and the secondary magnetic field is detected by another coil. MD can differ very much in their coil designs. The information content of the detected signal S(t) depends on the operational mode, that is, on the primary signal, which could be a pulse-like or harmonic. In the time-domain mode (pulse mode), the decline of S(t) is of primary interest. In the frequency mode, the amplitude and phase of S(t) are of primary interest..

The MD is extremely sensitive to even tiny metallic objects but, due to lack of differentiation capability, the MD suffers from high false-alarm rates caused by ubiquitous clutter./debris, as well as from the potentially strong dependence of its performance on the spatially often very heterogeneous electromagnetic properties (e.g. magnetic susceptibility) of the soil. There are different approaches of attacking such problems:

(a) Combi-detection: Various R&D projects (e.g. [2]) are focused on combinations of the MD with at least one complementary sensor that contributes near-orthogonal information. This could be either another anomaly sensor (e.g. ground penetrating radar) or a confirmation sensor which senses constituents of explosives (e.g. artificial nose).

(b) Signal analysis and pattern recognition: Use of general tools (e.g. machine learning, neural networks, wavelets) and specific EMI-related approaches (e.g. for time-domain EMI sensors, EMI spectroscopy with wideband frequency-domain [3]-[5]), for extraction and classification of mine-like target features and/or soil properties.

(c) 3D local imaging: EMI is used for imaging of subsurface sensing of electrical conductivity patterns, for example in exploration geophysics (e.g. [6],[7]) and non-destructive testing (e.g. [8]). Primarily designed for other spatial scales, subsurface imaging methodologies from these disciplines are potentially transferable to scales relevant in near-field mine detection.

The approaches (b) and (c) are followed parallely in the recently launched first phase of a new R&D program, funded by the German Federal Ministry of Education and Research (BMBF), on detector technological aspects of humanitarian demining. The approach (a), is postponed to a later R&D program phase, which is currently in conceptual planning, with advice from external experts.

2. 3D local imaging

With suitable operational scan procedures, some additional hardware (high-resolution monitoring of MD position and orientation; portable computation and visualization capacity) and suitable numerical algorithms, local EMI 3D imaging with a MD should be possible.

When the MD, operated in its normal mode, signals an anomaly, the operator could start a local EMI scan mode: automatic sampling (in small time intervals) of the momentary values of MD position, the MD orientation and the received secondary signal(s). This mode activated, the operator could perform a local MD scan in an area around the detected anomaly, varying not only the spatial position $\boldsymbol{r}\in\Re^3$ of the MD but also its spatial orientation $\boldsymbol{\Omega}\in\Re^3$ and other MD operational parameters (e.g. source frequency), contained in a parameter vector $\boldsymbol{p}\in\Re^p$. The EMI scan procedure will yield the following data for 3D local imaging: m signals $s_i(t)$ for configurations $\boldsymbol{c}_i = (\boldsymbol{r}_i,\boldsymbol{\Omega}_i,\boldsymbol{p}_i)$; $i = 1,...,m$.

Assuming a suitable operational procedure of the local EMI scan mode, that is, a suitable pattern of configuration point set $\{\boldsymbol{c}_1,...,\boldsymbol{c}_m\}$ within a finite volume of $\Re^{p+6}$, taking into account the anisotropy of the primary magnetic field (usually a magnetic dipole), and considering the good performance record of CT in various applications (e.g. medicine), the above-described goal of subsurface 3D imaging should be achievable.

The following step approach appears tempting, at first glance: (1) Choice of a pattern of local subsurface conductivities (2) With this pattern, simulation (incorporating knowledge on the MD model of interest) of the MD measurement process, for configurations $\boldsymbol{c}_1,...,\boldsymbol{c}_m$, yielding signals $s_1(t),...,s_m(t)$. (3) Comparison of predicted and measured signals. (4) Variation of the pattern of subsurface conductivities, aiming at a better matching of virtual and real signals (5) Restart with step 1 and iteration of the procedure. With this procedure, search of the pattern that gives the best match (e.g. least squares or maximum likelihood).

It is unlikely that such kind of brute-force approach will work in real-time (which is operationally mandatory). Fast and efficient alternatives are needed here. For this sake, mathematical expertise (e.g. from computer tomography) is involved in the program.

Performance parameters, such as computational speed, attainable spatial resolution and reliability, remain to be seen. Should the attainable reliability and spatial resolution turn out insufficient for mine detection it might suffice for unexploded ordnance (UXO). Interim results from the planned mathematical analyses, particularly from forward modelling, will upgrade the understanding of MD measuring processes and thus can give useful hints for fine-tuning of MD hardware (e.g. coil design).

3. Mathematical treatment

Major challenges of local EMI imaging with a MD are the forward modelling of measurement processes and the solution of inverse problems. Particularly useful in this context could be mathematical methods from exploration geophysics and from medical computer tomography. Precise identification of the relevant states of the art in relevant methodologies will be a priority task for the start phase of the program; the selected references given below are just for substantiating the expectation that the EMI 3D local imaging approach should be feasible.

3.1. Maxwell's equations

The main problem of forward modelling of the EMI measurement process is the complex dynamics of the interaction of several fields (each depending on both position $\boldsymbol{r}$ and time t): electric field intensity $\boldsymbol{E}$, magnetic field intensity $\boldsymbol{H}$; magnetic flux density $\boldsymbol{B}$; electric flux density $\boldsymbol{D}$; electric current density $\boldsymbol{j}$ and electric charge density ρ.

The macroscopic phenomena of classical electro-dynamics are described (up to the Lorentz force exerted by $\boldsymbol{B}$ on a moving charged particle) by Maxwell's equations. The related theory is reproduced below, for better illustration of the problem, from standard textbooks (e.g. [9]) on classical electrodynamics. In their differential form, Maxwell's equations read

$$\text{curl } \boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \tag{1}$$
$$\text{curl } \boldsymbol{H} = \boldsymbol{j} + \partial\boldsymbol{D}/\partial t \tag{2}$$
$$\text{div } \boldsymbol{B} = 0 \tag{3}$$
$$\text{div } \boldsymbol{D} = \rho \tag{4}$$

with partial time derivative $\partial/\partial t$ and vector differential operators **curl** and **div** which act on the space co-ordinates.

For a vector field $\boldsymbol{F}$, **div** $\boldsymbol{F}$ gives its divergence (source density), **curl** $\boldsymbol{F}$ gives its rotation (vortex density). One has **div** $\boldsymbol{F} = \nabla\cdot\boldsymbol{F}$ and **curl** $\boldsymbol{F} = \nabla\times\boldsymbol{F}$, formally scalar and vector products with the nabla operator ∇ which contains partial derivatives with respect to the space co-ordinates. The explicit definition of ∇ depends on the type of co-ordinate system (e.g. cartesian, spherical, cylindrical); e.g. [9]. Two central integral laws of vector differential calculus,

$$\int_{\partial V} \boldsymbol{F}\cdot d\boldsymbol{a} = \int_V \textbf{div } \boldsymbol{F} dv \quad \text{(Gauß)}$$
$$\int_{\partial A} \boldsymbol{F}\cdot d\mathbf{s} = \int_A \textbf{curl } \boldsymbol{F}\cdot d\boldsymbol{a} \quad \text{(Stokes)}$$

which hold for arbitrary surface A (with closed border line ∂A) and for arbitrary volume V (with closed border surface ∂V), respectively, link the differential form of Maxwell's equations to their integral form

$$\int_{\partial A} \boldsymbol{E}\cdot d\boldsymbol{s} = -d/dt \int_A \boldsymbol{B}\cdot d\mathbf{a} \quad (5)$$
$$\int_{\partial A} \boldsymbol{H}\cdot d\mathbf{s} = I + d/dt \int_A \boldsymbol{D}\cdot d\boldsymbol{a} \quad (6)$$
$$\int_{\partial V} \boldsymbol{B}\cdot d\boldsymbol{a} = 0 \quad (7)$$
$$\int_{\partial V} \boldsymbol{D}\cdot d\boldsymbol{a} = Q \quad (8)$$

with electric charge $Q = \int_V \rho dv$ and current $I = \int_A \boldsymbol{j}\cdot d\boldsymbol{a}$. Both forms of Maxwell's equations, (1)-(4) and (5)-(8), are equivalent. It depends on the actual problem which of them is more convenient for forward modelling.

As must be, Maxwell's equations imply the continuity equation for electric charge which reads (in case of no charge sources or sinks)

$$\mathbf{div}\, \boldsymbol{j} + \partial\rho/\partial t = 0$$

At a boundary surface A, Maxwell's equations imply the following boundary conditions:

$$\boldsymbol{n}\times\boldsymbol{\delta E} = 0$$
$$\boldsymbol{n}\times\boldsymbol{\delta H} = \boldsymbol{j}_A$$
$$\boldsymbol{n}\cdot\boldsymbol{\delta B} = 0$$
$$\boldsymbol{n}\cdot\boldsymbol{\delta D} = \rho_A$$

where ρ_A and $\boldsymbol{j}_A$ are the surface charge and flux densities, respectively, $\boldsymbol{\delta F}$ denotes the change of vector field $\boldsymbol{F}$ when passing through surface A, and $\boldsymbol{n}$ is the normal unit vector on surface A.

Maxwell's equations are complemented by material equations

$$\boldsymbol{B} = \mu\mu_0\boldsymbol{H} = \mu_0\boldsymbol{H} + \boldsymbol{M} \quad (9)$$
$$\boldsymbol{D} = \varepsilon\varepsilon_0\boldsymbol{E} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} \quad (10)$$
$$\boldsymbol{j} = \sigma\boldsymbol{E} \quad (11)$$

with universal electromagnetic constants ε_0 and μ_0. The material parameters ε (electric permittivity), μ (magnetic permeability) or σ (electric conductivity) may depend on $\boldsymbol{r}$ and t.

Additional material equations are $\boldsymbol{P} = \chi_e\boldsymbol{E}$, with electric susceptibility χ_e, and $\boldsymbol{M} = \chi_m\boldsymbol{H}$, with magnetic susceptibility χ_m (the latter relation holds for paramagnetic and diamagnetic materials only; with ferromagnetic materials, things are more complex). In case of isotropy, the material parameters are scalars, in case of anisotropy (e.g. crystals, permanent magnetization) they are tensors.

The solution of Maxwell's equations can be facilitated by the introduction of time-dependent potentials, the scalar potential Φ and the vector potential $\boldsymbol{A}$, with

$$\boldsymbol{E} = \mathbf{grad}\, \phi - \partial\boldsymbol{A}/\partial t$$
$$\boldsymbol{B} = \mathbf{curl}\, \boldsymbol{A}$$

where **grad** denotes the gradient. $\boldsymbol{A}$ and Φ are defined up to a gauge transformation. The Coulomb (or transversal, or radiation) gauge yields $\mathbf{div}\, \boldsymbol{A} = 0$. The Lorentz gauge yields $\mathbf{div}\, \boldsymbol{A} + \varepsilon\mu\partial\Phi/\partial t = 0$. With the latter, one ends up with the inhomogeneous wave equations

$$(\nabla^2 - \varepsilon\mu\partial^2/\partial t^2)\phi = -\rho$$
$$(\nabla^2 - \varepsilon\mu\partial^2/\partial t^2)\boldsymbol{A} = -\boldsymbol{j}$$

where Laplace operator ∇^2, formally a scalar product of operator $\boldsymbol{\nabla}$ with itself, contains second order partial derivatives with respect to the spatial co-ordinates.

Powerful tool for solution of wave equations of the form and the like are Green's functions. A Green function to the operator $(\nabla^2 + \varepsilon\mu\partial^2/\partial t^2)$ satisfies, in a given region of $\Re^3$,

$$(\nabla^2 - \varepsilon\mu\partial^2/\partial t^2)G(\boldsymbol{r},t;\boldsymbol{r}',t') = \delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')$$

with the Dirac delta distribution δ, and fulfills certain conditions on the boundaries of this region. Such conditions can be posed directly on the values of G (Dirichlet type) or on the values of the normal derivative ∂G/∂n (Neumann type) on the boundary; the Dirichlet–to–Neumann map relates the solution on the boundary to its normal derivative on the boundary. With Green's functions, solutions to the wave equations

$$(\nabla^2 - \varepsilon\mu\partial^2/\partial t^2)\Psi(\boldsymbol{r},t) = f(\boldsymbol{r},t)$$

are usually obtained in the form of convolution integrals, with kernel function G. In infinite space and with no boundary conditions, the solution is of the form

$$\Psi(\boldsymbol{r},t) = \int G(\boldsymbol{r},t;\boldsymbol{r}',t')f(\boldsymbol{r}',t')dv'dt'$$

1.2. Forward modelling

Forward modelling is necessary for relating the detected signal s(t) to the MD generated source field, which depends on the above-defined measurement configuration $\boldsymbol{c} = (\boldsymbol{r},\boldsymbol{\Omega},\boldsymbol{p})$. The source field generated by the MD must be known as an input (for preliminary purposes, particularly for tests and demonstrations of computation performance in the early project phases, the well-known magnetic field generated by a current in a circular planar coil may be assumed).

Forward modelling by solution of Maxwell's equations (or of derivates of them), for given start an boundary conditions, can be a difficult task. For non-simplistic, realistic measurement geometries, analytical solution in closed form will not be possible; numerical treatment will become necessary.

The transfer of Maxwell equations (in their differential of integral form) into algebraic forms requires some kind of discretization (e.g. finite differences, finite elements, finite volumes) on a suitable, not necessarily regular mesh. There is some literature (e.g. [10]-[13]) on this subject (also on discretizations that conserve, as much as possible, the underlying physics). Special expertise (e.g. [14]) on consistent discretization of Maxwell's equations (in their integral form, on a dual grid, using a finite integration technique) is considered for contribution to the program.

Forward modelling of the MD measurement process gains quantitative understanding of the physics behind it. Such understanding will help coping with a serious problem: the heterogeneity of the electromagnetic properties of soils and their influence on MD performance (e.g.[15]). The potential benefits of forward modelling are not restricted to EMI 3D local imaging but will also extend to signal analysis.

1.3. Inverse problems

Reconstruction of 3D subsurface patterns of (primarily) electric conductivity, on the empirical basis of data set S (see above) and on the theoretical basis of forward models, constitutes an inverse problem (e.g.[16]). Inverse problems are also part of computer tomography (CT) for medical diagnostic imaging (e.g. [17]).

Inverse problems are often ill-posed in the sense that small errors in the data may cause very large errors in the reconstructed solutions. A potential mitigation tool is regularization (e.g. [18]), that is, stabilization of the solution by explicit incorporation of additional a-priori information into the approach. A potentially useful type of a-priori information is signature (of mines, of soils); this links EMI 3D local imaging and MD signal analysis.

A major problem is that inverse problems, particularly those in the EMI imaging context, tend to be highly non-linear, with the associated problems of non-uniqueness of solution, not to speak of the impact on computation time.

1.4. Inverse scattering

There are various approaches to the problems described above (e.g. [19]-[23]). There is some expertise in Gemany on certain approaches (e.g.[23]-[30]) which are well tested for the scalar Helmholtz equation $(\nabla^2 + k^2)\Psi = f$, with k = ω/c (which results when the search for solutions of the scalar wave equation $(\nabla^2 - \varepsilon\mu\partial^2/\partial t^2)\Psi = f$ is restricted to standing wave harmonics with angular frequency ω). Due to their potential, these approaches (which differ in the way the data errors are treated and in the information they provide) are considered for adaptation to the inverse problem of Maxwell's equations and are sketched below.

One approach, employing the linear sampling method (e.g. [24]]) for the Helmholtz equation and extended (e.g. 25]) to the inverse problem for the potential equation and to electric impedance tomography, is based on a factorization of the Dirichlet–to–Neumann map, relating the solution on the boundary to its normal derivative. The ranges of different operators are characterized and shown to be the same for suitably chosen operators. The original nonlinear problem described by the Dirichlet–to–Neumann map is thus reduced to the study of the range of a linear operator whose singular value decomposition can be computed for homogeneous background. The method works as follows. For each point $z \in \Re^3$ the boundary values of a dipole potential h_z is determined. The point z is inside the domain D of the metallic object if and only if it lies in the range of the above mentioned operator; i.e., the Picard criterion for h_z is fulfilled, which means a certain series converges. This series is computed via the singular value decomposition and convergence is checked with the help of the decay of the computed expansion coefficients. The data errors enter these coefficients. Hence for each point in the examined region the information whether this point belongs to a metallic body or not is provided, hence an image in produced. From its shape one can conclude whether a mine is in this examined region or not.

A second approach (e.g. [23],[26]) is based on results on the dependence of the measured field from the boundary of the searched–for object [23]. The nonlinear problem is solved by variants of Newton's method; i.e., for a given initial guess the problem is linearized and corrections are computed. With a priori assumptions the number of iterations can be kept small. The regularization of the data errors is achieved by choosing a suitable mesh size which must no be too small. In that way the boundary of the metallic object is determined. The method is well established for the Helmholtz equation and has the potential to be extended to the problem under consideration

A third approach (e.g. [27]},[28]) is based on the separation of the ill–posed nonlinear problem into a linear ill–posed problem and a well–posed nonlinear problem (e.g. [28]). By way of example, this approach is sketched here in more detail. The product Ψ of the searched for object function f and the field u is determined here as the solution of the integral formulation of Maxwell's equations. In the scalar case of the Helmholtz equation this reads as

$$u^s(\alpha,x) = \int G(k,x\text{-}y)\,\Psi(\alpha,y)\mathrm{d}y \qquad (12)$$

with $\Psi(\alpha,y) = u(\alpha,y)f(y)$. This solution step is linear, it can be solved by the method of approximate inverse [29], where a reconstruction kernel is precomputed independently of the data. The method is of filtered back-projection type like the fast methods in x-ray CT. In this step well established methods for noise reduction are applied by filtering the data.

The drawback is that for each measurement for varying parameter α the linear problem has a large null space. (It is to be studied if these reconstructions are sufficient to be the input of signal analysis, particularly for classification algorithms like support vector machines.)

For the imaging the null space can be explicitly characterized, and this information enters in a nonlinear way into the second step of the reconstruction method. Thereby the influence of so–called ghost solutions can be reduced sufficiently. This more expansive step is only necessary if the first reconstructions indicate some metallic parts scattered in the region under consideration. This can be done in an adaptive way. Images of the ground are thus provided, which have to be analyzed accordingly. The method is shown to work for the Helmholtz equation; first results for the Maxwell equation are available.

2. Integrated approach

The above-described R&D problem, with its many facets, requires a co-ordinated, approach which integrates expertise from various scientific disciplines, such as computational electrodynamics, numerical mathematics, computer tomography, signal analysis, applied informatics, exploration geophysics and soil science. Advice from MD manufacturers and MD users is also mandatory. This will be implemented in the German R&D program. As described above there are two main fields of research, which are followed in parallel: EMI 3D imaging and signal analysis. Forward modeling of the MD measurement process is fundamental for EMI 3D imaging but also useful for signal analysis.

The aim of the forward modeling is to provide computational simulations of the MD measurement process and thus to promote the quantitative understanding of the physics behind. Realistic modelling must be capable of mapping the complex influence of soil properties on MD performance and thus needs the relevant input data from MD technology and from soil geophysics.

In the early phases of the program, the signal analysis R&D line aims at the discrimination of different types of objects, on the basis of MD raw data. Results of this discrimination may provide useful a-priori information for the regularization of inverse problems in 3D imaging. In the later phases of the project signal analysis methods can be applied to 3D imaging data provided by solutions of inverse problems.

The integration of these activities goes into the direction of the long-propagated (e.g. [29]) unification of subsurface sensing and imaging systems. In order to ensure a high efficiency of this inter-disciplinary research project a co-ordinated approach is required. A framework is set up which supports and encourages the collaboration between the main areas of research.

In order to achieve a high transparency for all members of the project and to monitor the progress the research will be accompanied by regular assessments of attainable performance (e.g. reliability, computational speed, spatial resolution). Besides the internal mutual data supply (e.g. simulations of the MD measurement process, experimental measurement) there is an additional need of experimental data acquired under realistic conditions.

This demands for co-ordinated co-operation of the involved scientists with MD manufacturers and MD users (deminers). The developed methods will be tested, under realistic conditions, in close co-operation with the International Test and Evaluation Program (ITEP).

The term of the first program phase will be three years. Priority task of the start phase of the project is a precise identification of the states of the art in the relevant methodologies. During the first one to two years, the feasibility of the new approaches and their interaction potential will regularly be checked, following a staggered time schedule. In the remaining time, the main focus will be on the implementation of the results.

It remains to be seen if the attainable resolution and reliability of the approaches to 3D local imaging will be sufficient for implementation in practical MD operations. Whatever will come out, it is expected that many interim results and their contribution to a better understanding of the physics of MD based sensing will be useful in future R&D programs on mine detection technology for humanitarian demining.

Acknowledgment

Prior to the launch of the above-described first phase of the R&D funding program on mine detection technologies for humanitarian demining, the concept has been discussed by BMBF with multi-disciplinary boards of experts from MD-relevant areas in science, technology and application.

3. References

[1] C. Bruschini: "A Multidisciplinary Analysis of Frequency Domain Metal Detectors for Humanitarian Demining", Ph.D. thesis, Faculty of Applied Sciences, Vrije Universiteit Brussel, 2002.

[2] Eudem 2: Currently funded or previous EU projects Available: http://www.eudem.vub.ac.be/projects/

[3] Ping Gao, L. Collins, P.M. Garber, N. Geng, L. Carin, "Classification of Landmine-Like Metal Targets Using Wideband Electromagnetic Induction", *IEEE Transactions on Geosciences and Remote Sensing*, Vol. 38, pp. 1352-1361, 2000.

[4] I.J. Won, D.A. Keiswetter, T.A. E. Novikova, "Electromagnetic Induction Spectroscopy", *Journal of Environmental and Engineering Geophysics*, Vol. 3, pp. 27-40, 1998.

[5] S.L. Tantum, L.M. Collins, "A Comparison of Algorithms for Subsurface Target Detection and Identification Using Time-Domain Electromagnetic Data", *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 39, pp. 1299-1306, 2000.

[6] J.T. Weaver, *Mathematical Methods for Geo-electromagnetic Induction*. Research Studies Press, Taunton, 1994.

[7] M.S. Zhdanov, *Three-Dimensional Electromagnetics* (= *Methods in Geophysics*, Vol. 35), Society of Exploration Geophysicists, Tulsa, 2002.

[8] J. Blitz, *Electrical and Magnetic Methods of Non-Destructive Testing*, Chapman and Hall, London, 1997.

[9] J.D. Jackson, *Classical Electrodynamics*, 3rd edition, Wiley, New York Chichester, 1999.

[10] D.A. Aruliah, U.M. Ascher, E. Haber, D. Oldenburg, "A Method for the Forward Modelling of 3-D Electromagnetic Quasi-static Problems", *Mathematical Models and Methods in the Applied Sciences*, Vol. 11, pp. 1-21, 2001.

[11] P. Monk, *Finite Element Methods for Maxwell's Equations*, Clarendon Press / Oxford University Press, Oxford, 2003, 496 pp.

[12] J.M. Hyman, M. Shashkov, "Mimetic Finite Difference Methods for Maxwell's Equations and the Equations of Magnetic Diffusion", *Progress in Electromagnetic Research (PIER)*, Vol. 32, pp. 89-121, 2001.

[13] S, Piperno, M. Remaki, L. Fezoui, "A Nondiffusive Finite Volume Scheme for the Three-dimensional Maxwell Equations on unstructured Meshes", *SIAM Journal of Numerical Analysis*, Vol. 39, pp. 2089-2108, 2002.

[14] U. van Rienen, *Numerical Methods in Computational Electrodynamics* (= *Lecture Notes in Computational Science and Engineering*, Vol. 12), Springer, Berlin Heidelberg New York, 2001.

[15] International Test and Evaluation Program []ITEP]: Proceedings of the Discussion Day on Soil Electromagnetic Characteristics and Metal Detector Performance, EU-JRC, Ispra, 2002. Available: http://www.itep.ws/whatsnew/121202/soil.html.

[16] D. Colton, H.W. Engl, A.K. Louis, J.R. McLaughlin, W. Rundell (eds.), *Surveys on Solution Methods for Inverse Problems*, Springer, Wien New York, 2000.

[17] Natterer, "Numerical Methods in Tomography", *Acta Numerica*, Vol. 8, pp. 1-35, 1999.

[18] L. Tenorio, "Statistical Regularization of Inverse problems" , *SIAM Reviews*, Vol. 43, pp. 347-366, 2001.

[19] O. Dorn, H. Bertete-Aguirre, J.G. Berryman, G.C. Papanicolaou, "A Nonlinear Inversion Method for 3D Electromagnetic Imaging Using Adjoint Fields", *Inverse Problems*, Vol. 15, pp. 1523-1558, 1999.

[20] G. Xie, J. Li, E.L. Majer, D. Zuo, M.L. Oristaglio, "3-D Electromagnetic Modeling and Nonlinear Inversion", *Geophysics*, Vol. 65, pp. 804-822, 2000.

[21] M. Zhdanov, E. Tartaras, "Three-dimensional Inversion of Multitransmitter Electromagnetic Data Based on the Localized Quasi-linear Approximation, *Geophysics Journal International*, Vol. 148, pp. 506-519, 2002.

[22] D. Colton, H. Haddar, P. Monk: "The Linear Sampling Method for Solving the Electromagnetic Inverse Problem". *SIAM Journal on Scientific Computing*, Vol. 24, pp. 719-731, 2002.

[23] D. Colton, R. Kress: *Inverse Acoustic and Electromagnetic Scattering Theory*, 2nd edition. Springer, New York, 1998.

[24] A. Kirsch, P. Monk, "A Finite Element Method for Approximating Electromagnetic Scattering from a Conducting Object", *Numerische Mathematik*, Vol. 92, pp. 501-534, 2002.

[25] M. Hanke, M. Brühl, "Recent Progress in Electric Impedance Tomography", *Inverse Problems*, in the press (2003).

[26] R. Potthast, *Point sources and Multipoles in Inverse Scattering*, Chapman and Hall, London, 2001.

[27] H. Abdullah, A.K. Louis, "The Approximate Inverse for Solving an Inverse Scattering problem for Acoustic Waves in an Inhomogeneous Medium", *Inverse Problems*, Vol. 15, pp. 1213-1229, 1999.

[28] A.K. Louis: "Approximate Inverse for Linear and Some Nonlinear Problems". *Inverse Problems*, Vol. 12, pp. 175-190, 1996.

[29] M.B. Silevitch, S.W. McKnight, C. Rappaport, "A Unified iscipline if Subsurface Sensing and Imaging systems", *Subsurface Sensing Technologies and Applications*, Vol. 1, pp. 3-21, 2000.