Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

Calculus of Variations (Summer Term 2017)
Assignment H2 - Homework

Problem 2.1 (10 Points)

A brachistochrone is required to be constructed from a given curve $g_1(x, y) = 0$ to a second given curve $g_2(x, y) = 0$. What relationships must sutisfy the given curves at the respective points of intersection with the brachistochrone?

Problem 2.2 (5 Points)

Suppose that, in the solution of a specific isoperimetric problem, computation of the Lagrange multiplier yields the result $\lambda = 0$. what is the significance of this result?

Problem 2.3 (5 Points)

Given that

$$J[y] = \int_0^1 \left(y'^2 - y^2\right) dx$$

subject to the constraint

$$\int_0^1 \sqrt{1 + y'^2} dx = \sqrt{2},$$

and the end conditions $y(0) = 0$ and $y(1) = 1$. Prove that $J[y]$ achieves its minimum value for $y = x$.

Problem 2.4 (5 Points)

Consider the functional

$$J[y, z] = \int_{x_0}^{x_1} \left(y^2 + z^2\right) dx$$

subject to the constraint

$$y' = z - y.$$

What type of the constaint do we have? Write down the form of the problem including a Lagrange multiplier in the integral. Determine the Euler-Lagrange equations for y and z. Solve the equations to find the form of the extremal curve of J under the constraint.

Deadline for submission: Friday, June 02, 08:30 am