Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

Calculus of Variations (Summer Term 2017)
Assignment H3 - Homework

Problem 3.1 (4+4=8 Points)

Find the form of extremals of the following functionals

a) $$J_1[y,z] = \int_{x_0}^{x_1} \left(2yz - 2y^2 + y'^2 - z'^2\right) dx$$

b) $$J_1[q_1,q_2,q_3] = \int_{t_1}^{t_2} \left(\dot{q}_1\dot{q}_2 + \dot{q}_2 q_3 + \dot{q}_3 q_1\right) dt$$

Problem 3.2 (5 Points)

Consider the functional

$$J[y,z] = \int_{x_0}^{x_1} \left(y^2 + z^2\right) dx$$

subject to the constraint

$$y' = z - y.$$

What type of the constaint do we have? Write down the form of the problem including a Lagrange multiplier in the integral. Determine the Euler-Lagrange equations for y and z. Solve the equations to find the form of the extremal curve of J under the constraint.

Problem 3.3 (5 Points)

The Beltrami identity states that the extremal function of the integral

$$I[u] = \int_a^b L(x,u,u')dx$$

satisfies the differential equation

$$\frac{d}{dx}\left(L - u'\frac{\partial L}{\partial u'}\right) - \frac{\partial L}{\partial x} = 0.$$

Please prove the identity using the Euler-Lagrange equation and the chain rule. Note that as a special case, when L does not depend on x, we get the equation for the autonomous case, i.e., $H = const.$

Problem 3.4 (8 Points)

Use Ritz's method to find an approximate, non-trivial solution to the differential equation

$$y'' + \frac{1}{x}y' + \lambda y = 0,$$

in the domain $x \in [0, 1]$ where $y(0)$ is non-singular and $y(1) = 1$, and hence determine an approximate value of λ that has a solution.

[HINTS: not that the equation can be written in the form

$$\frac{d}{dx}(xy') + \lambda xy = 0,$$

and find the corresponding integral for which this is the Euler-Lagrange equation.

Once you have a variational problem, use the trial function

$$y_{trial} = a + bx^2 + cx^4,$$

which we have chosen because the solution is expected to be an even function.]

Deadline for submission: Wednesday, June 16, 08:30 am