Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

Calculus of Variations (Summer Term 2017)
Assignment H4 - Homework

Problem 4.1 (8 Points)

Minimize the functional

$$J[y] = \int_0^2 (y' + 1)^2 (y')^2 dx$$

subject to the end-point conditions that $x(0) = 1$ and $x(2) = 0$.

(*Hint:* consider the possibilty of broken extremals).

Problem 4.2 (3x3=9 Points)

Decide which of the following functionals admit broken extremals or not:

$$J_1[y] = \int_0^1 \left(y'^6 - 5y'^4 + 15y'^2\right) dx, \qquad J_2[y] = \int_0^1 y'^3 dx,$$

$$J_3[y] = \int_0^1 \cos\{(y')\}\, dx.$$

(Either construct examples of broken extremals or rule out their existence unsing available theory). Moreover, determine at least one admissible extremal for the minimization of each of the functionals over the set of functions y satisfying $y(0) = 0$ and $y(1) = \pi/2$.

Problem 4.3 (8 Points)

Use Rits's method to find an approximate solution to minimize the

$$J[y] = \int_0^{2\pi} \left(y'^2 + \lambda^2 y^2\right) dx, \qquad y(0) - 0, \quad y(1) = 1$$

where $y(0) = 1$ and $y(2\pi) = 1$ and λ is a positive integer. Use the trial functions

$$\phi_n(x) = \cos(nx).$$

Compare your solution to one found directly from the Euler-Lagrange equations.

Problem 4.4 (9 Points)

Minimize the functional

$$J[y] = \int_{-1}^{1} y\sqrt{1 + (y')^2}dx$$

subject to the end-point conditions that $y(-1) = 2$ and $y(1) = 2$. Use the modification of the Ritz Method that we discussed in Lecture 13. Compare the obtained result with the result from Lecture 13.

Hint:

1. Approximate a curve by a polynomial
$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$$
2. Calculate $J[a_i]$ and corresponding derivatives $\frac{\partial J}{\partial a_i}$.
3. Calculate (using Maple) the roots a_i provided $\frac{\partial J}{\partial a_i} = 0$.

Problem 4.5 (7 Points)

Minimize

$$J[u] = \int_{0}^{1} u^2 dt$$

subject to

$$\dot{x}_1 = u - x_2$$
$$\dot{x}_2 = -u$$

and

$$x_1(0) = 2$$
$$x_1(1) = 1$$
$$x_2(0) = 0$$
$$x_2(1) = 1$$

Deadline for submission: Friday, June 30, 08:30 am