Üb 12

1) (i) $\int_0^1 (4\sqrt{x} - 2x^3)\,dx = 4\int_0^1 \sqrt{x}\,dx - 2\int_0^1 x^3\,dx =$

$= 4\, x^{3/2}\, \frac{2}{3}\Big|_0^1 - 2\,\frac{x^4}{4}\Big|_0^1 = \frac{8}{3} - \frac{1}{2} = \frac{13}{6}$

(ii) $\int_{-1}^{1} e^{-3x}\,dx = \int_{-3}^{3} e^{-y}\,\frac{1}{3}\,dy = \frac{1}{3}(-e^{-y})\Big|_{-3}^{3} = \frac{e^{3} - e^{-3}}{3}$

(iii) $\int_1^2 \left(\frac{1}{x} + \frac{1}{x-3}\right) dx = \log|x|\Big|_1^2 + \log|x-3|\Big|_1^2 =$

$= \log 2 - \log 2 = 0$

(iv) $\int_9^{16} x^{-3/2}\,dx = -2x^{-\frac{1}{2}}\Big|_9^{16} = -\frac{1}{2} + \frac{2}{3} = \frac{1}{6}$

5 $\int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = 2$

$\int_0^{\pi} |\cos x|\,dx = \int_0^{\frac{\pi}{2}} \cos x\,dx - \int_{\frac{\pi}{2}}^{\pi} \cos x\,dx = 2$

2) $\int_{-\pi}^{\pi} x \sin x\,dx = -x\cos x\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \cos x\,dx = 2\pi$

$\int_0^1 x^2 e^x\,dx = x^2 e^x\Big|_0^1 - \int_0^1 2xe^x = e - 2xe^x\Big|_0^1 + \int_0^1 2e^x\,dx =$

$= -e + 2e - 2 = e - 2$

$(y \log y)' = \log y + 1$

$$\int_0^2 x \log(x^2+1)\,dx = \left| \begin{matrix} y = x^2+1 \\ y' = 2x \end{matrix} \right| = \int_1^5 \frac{1}{2} \log y \, dy =$$

$$= \frac{1}{2} \left[y \log y \Big|_1^5 - \int_1^5 y \frac{1}{y} dy \right] = \frac{1}{2}(5 \log 5 - 4)$$

$$\int_0^1 \sqrt{1-x^2}\,dx = \left| \begin{matrix} x = \sin y \\ x' = \cos y \, y' \end{matrix} \right| = \int_0^{\frac{\pi}{2}} \cos y \cdot \sqrt{1-\sin^2 y}\, dy =$$

$$= \int_0^{\frac{\pi}{2}} \cos^2 y \, dy = \frac{\pi}{2} - \underbrace{\int_0^{\frac{\pi}{2}} \sin^2 y \, dy}_{} = \frac{\pi}{4}$$

$\leftarrow$ gabs im Vorlesung

A.3:

$$\int_0^3 (1-x)e^{-x} dx = \int_0^3 e^{-x} dx - \int_0^3 x e^{-x} dx = e^{-3} - 1 + x e^{-x} \Big|_0^3 - \int_0^3 e^{-x} dx$$

$$= 3e^{-3}$$

$\int_1^5 \sqrt{2t-3}\, dt$ existiert nicht da die Funkt.
$f(t) = \sqrt{2t-3}$ ist nur für $t \geq \frac{3}{2}$ definiert.

$$\int_{-1}^{1} \frac{u+1}{u^2+2u+4} du = \left| \begin{matrix} x(u) = u^2+2u+4 \\ x'(u) = 2(u+1) \end{matrix} \right| = \int_3^7 \frac{1}{2} \frac{dx}{x} =$$

$$= \frac{1}{2}(\log 7 - \log 3)$$

$$\int_0^1 \log \sqrt{2v+1}\, dv = \frac{1}{2} \int_0^1 \log(2v+1)\, dv = \left| \begin{matrix} x = 2v+1 \\ x' = 2 \end{matrix} \right| =$$

$$= \frac{1}{4} \int_1^3 \log x \, dx = \frac{1}{4} \int_1^3 (x \log x)' - 1 \, dx = \frac{1}{4}\left[3 \log 3 - 2\right]$$

A4: a) $F_1(y) = \int_2^y \frac{x+5}{(x+3)(x-1)} dx$.

$$\frac{1}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} \Rightarrow A(x-1)+B(x+3) = 1$$
$$x(A+B) - A + 3B = 1$$

$$\Rightarrow \begin{cases} A+B=0 \\ 3B-A=1 \end{cases} \begin{cases} A=-B \\ 4B=1 \end{cases} \begin{cases} A=-\frac{1}{4} \\ B=\frac{1}{4} \end{cases}$$

$$F_1(y) = \int_2^y \frac{x+5}{4(x-1)} - \frac{x+5}{4(x+3)} dx = \frac{1}{4}\int_2^y \frac{x-1+6}{x-1} - \frac{x+3+2}{x+3} dx =$$

$$= \frac{1}{4}\int_2^y 1 + \frac{6}{x-1} - 1 - \frac{2}{x+3} dx = \frac{1}{4}\left[6\log(y-1) - 2\log(y+3) + 2\log 5\right] =$$

$$= \frac{1}{2}\left[3\log(y-1) - \log(y+3) + \log 5\right]$$

b) $F_2(z) = \int_0^z \frac{1}{1+e^x} dx = \left| \begin{matrix} y(x) = e^x \\ y'(x) = e^x \end{matrix} \right| = \int_1^{e^z} \frac{1}{y(1+y)} dy =$

$$\frac{1}{y(1+y)} = \frac{A}{y} + \frac{B}{1+y} \Rightarrow y(A+B) + A = 1 \Rightarrow A=1,\ B=-1$$

$$= \int_1^{e^z} \frac{1}{y} - \frac{1}{1+y} dy = \log e^z - \log(1+e^z) + \log 2 =$$

$$= z - \log(1+e^z) + \log 2$$