

**PDE and Boundary-Value Problems (Winter Term 2015/2016)**  
**Assignment H3 - Homework**

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**Problem 3.1 (Transformation of IBVP - 6 Points)**

Transform

$$\text{PDE: } u_t = u_{xx}, \quad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs: } \begin{cases} u(0, t) = 0 \\ u(1, t) = 1 \end{cases}, \quad 0 < t < \infty$$

$$\text{IC: } u(x, 0) = x^2, \quad 0 \leq x \leq 1$$

to zero BCs and solve the new problem. What is the steady-state solution?

**Problem 3.2 (Solving the IBVP - 8 Points)**

Solve the problem

$$\text{PDE: } u_t(x, y, t) = u_{xx}(x, y, t) + u_{yy}(x, y, t), \quad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t < \infty$$

$$\text{BCs: } \begin{cases} u_x(0, y, t) = 0 \\ u_x(1, y, t) = -u(1, y, t) \\ u(x, 0, t) = 0 \\ u(x, 1, t) = 0 \end{cases} \quad 0 < t < \infty$$

$$\text{IC: } u(x, y, 0) = \left(1 - \frac{x^3}{3}\right) y(1 - y), \quad 0 < x < 1, \quad 0 < y < 1$$

by using the method of separation of variables.

**Problem 3.3 (Laplace Transform - 6 Points)**

Solve the problem

$$\text{PDE: } u_t = u_{xx}, \quad 0 < x < \infty, \quad 0 < t < \infty$$

$$\text{BC: } u(0, t) = \sin(t), \quad 0 < t < \infty$$

$$\text{IC: } u(x, 0) = 0, \quad 0 \leq x < \infty$$

by means of the Laplace transform (transform  $t$ ). What is the physical interpretation of this problem?

**Deadline for submission:** Monday, November 30, 10:15 am