

PDE and Boundary-Value Problems (Winter Term 2015/2016)
 Assignment H7 - Homework

Problem 7.1 (The Exterior Dirichlet Problem - 10 Points)

What is the solution to the exterior Dirichlet problem

$$\begin{array}{ll} \text{PDE} & \Delta u = 0, \\ \text{BC} & u(1, \phi) = 1 + \cos \phi \end{array} \quad 1 < r < \infty$$

Problem 7.2 (The Dirichlet Problem- 8 Points)

Solve the Dirichlet problem

$$\begin{array}{ll} \text{PDE} & \Delta u = 0, \\ \text{BCs} & \begin{cases} u(1, \theta) = \cos \theta \\ u(2, \theta) = \sin \theta \end{cases} \end{array} \quad 1 < r < 2$$

Problem 7.3 (Finite-Difference Method - 10 Points)

Solve the Neumann problem inside the square

$$\begin{array}{ll} \text{PDE} & \begin{cases} u_{xx} + u_{yy} = 0, & 0 < x < 1, \quad 0 < y < 1, \\ u = 0, & \text{On the top, bottom, and left-hand side of the square,} \\ \frac{\partial}{\partial x} u(1, y) = 1, & 0 \leq y \leq 1 \end{cases} \\ \text{BC} & \end{array}$$

by the finite-difference method?

Problem 7.4 (Questionnaire - 10 Points)

1. The equation $u_{tt} = u_{xx}$ is commonly called the _____ equation.
2. The term u_x in the equation $u_t = Du_{xx} - \nu u_x$ is related to the _____ of the material.

3. The Legendre polynomials $\{P_n(x)\}$ are _____ on the interval $[0, 1]$.
4. In order for separation of variables to work, the PDE must be _____ and _____.
5. An integral transform that we generally use on the time variable is due to _____.
6. The normal derivative at the boundary of a region is related to the _____ of material across the boundary.
7. The ODE in r we must solve when solving the vibrating circular membrane is due to _____.
8. A useful way to solve the equation $u_t = Du_{xx} - \nu u_x$ is by _____ coordinates.
9. The PDE $r^2 R'' + rR' - \lambda^2 R = 0$ is called _____ equation.
10. The PDE $u_{xx} + u_{yy} = f(x, y)$ is named after _____.

Deadline for submission: Monday, February 8, 10:15 am