Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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UNIVERSITÄT DES SAARLANDES

PDE and Boundary-Value Problems (Winter Term 2016/2017)
Assignment H2 - Homework

Problem 2.1 (Formulation of IBVP - 4x2=8 Points)

a) Suppose a laterally insulated metal rod of length $L = 1$ has an initial temperature of $\sin(3\pi x)$ and has its left and right ends fixed at temperatures zero and $10°C$. What would be the IBVP that describes this problem?

b) Suppose a metal rod laterally insulated has an initial temperature of $20°C$ but immediately thereafter has one end fixed at $50°C$. The rest of the rod is immersed in a liquid solution of temperature $30°C$. What would be the IBVP that describes this problem?

Problem 2.2 (Derivation of the diffusion equation - 6 Points)

Suppose $u(x,t)$ measures the concentration of a substance in a moving stream (moving with velocity ν). Suppose the concentration $u(x,t)$ changes both by diffusion and convection; derive the equation

$$u_t = \alpha^2 u_{xx} - \nu u_x$$

from the fact that at any instant time, the total mass of the material is not created or destroyed in the region $[x, x + \Delta x]$.

HINT: Write the conservation equation

$$\text{Change of mass inside } [x, x+\delta x] = \text{Change due to } \textit{diffusion} \text{ across the boundaries} \\ + \text{ Change due to the material being } \textit{carried} \text{ across the boundaries.}$$

Problem 2.3 (Interpretation of IBVP - 6 Points)

What is your interpretation of the IBVP?

$$\text{PDE:} \quad u_t = \alpha^2 u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u_x(1,t) = 1 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = \sin \pi x, \qquad 0 \leqslant x \leqslant 1$$

Can you draw rough sketches of the solution for different values of time? Will the solution come to a steady state; is this obvious?

Problem 2.4 (Units of quantities - 4 Points)

Substitute the units of each quantity into the equation

$$u_t = \alpha^2 u_{xx} - \nu u_x,$$

where ν has units of velocity to see that every term has the same units.

Deadline for submission: Wednesday, November 16, 10:15 am