Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
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PDE and Boundary-Value Problems (Winter Term 2016/2017)
Assignment H3 - Homework

Problem 3.1 (Solving IBVP - 4x3=12 Points)

a) What is the solution to the IBVP

$$\text{PDE:} \quad u_t = u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = 1, \qquad 0 \leqslant x \leqslant 1$$

(Note that this problem is physically impossible, since we are pulling the temperature from 1 to 0 instantaneously. In most problems, if the BCs are zero, then the initial themperature $\phi(x)$ should also be zero at $x = 0$ and $x = 1$).

b) What is the solution to problem a) if the IC is changed to

$$u(x,0) = \sin(2\pi x) + \frac{1}{3}\sin(4\pi x) + \frac{1}{5}\sin(6\pi x)?$$

c) What is the solution to problem a) if the IC is changed to

$$u(x,0) = x - x^2?$$

Problem 3.2 (Transformation of IBVP - 6 Points)

Transform

$$\text{PDE:} \quad u_t = u_{xx}, \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases}, \qquad 0 < t < \infty$$

$$\text{IC:} \quad u(x,0) = x^2, \qquad 0 \leqslant x \leqslant 1$$

to zero BCs and solve the new problem. What is the steady-state solution?

Problem 3.3 (Solving the IBVP - 8 Points)

Solve the problem

$$\text{PDE:}\quad u_t(x,y,t) = u_{xx}(x,y,t) + u_{yy}(x,y,t), \qquad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t < \infty$$

$$\text{BCs:}\quad \begin{cases} u_x(0,y,t) = 0 \\ u_x(1,y,t) = -u(1,y,t) \\ u(x,0,t) = 0 \\ u(x,1,t) = 0 \end{cases} \qquad 0 < t < \infty$$

$$\text{IC:}\quad u(x,y,0) = \left(1 - \frac{x^3}{3}\right) y(1-y), \qquad 0 < x < 1, \quad 0 < y < 1$$

by using the method of separation of variables.

Deadline for submission: Wednesday, November 30, 10:15 am