Universität des Saarlandes
Fachrichtung 6.1 – Mathematik
PD Dr. Darya Apushkinskaya

UNIVERSITÄT DES SAARLANDES

PDE and Boundary-Value Problems (Winter Term 2016/2017)
Assignment H5 - Homework

Problem 5.1 (Duhamel's Principle - 4 Points)

Using Duhamel's principle, what is the solution of the IBVP

$$\begin{aligned}
&\text{PDE:} \quad u_t = \alpha^2 u_{xx}, && 0 < x < 1, \quad 0 < t < \infty \\
&\text{BCs:} \quad \begin{cases} u(0,t) = 0 \\ u(1,t) = \sin(t) \end{cases}, && 0 < t < \infty \\
&\text{IC:} \quad u(x,0) = 0, && 0 \leqslant x \leqslant 1
\end{aligned}$$

Problem 5.2 (Convection Problem - 5 Points)

Solve the following convection problem:

$$\begin{aligned}
&\text{PDE:} \quad u_t = 2u_x, && -\infty < x < \infty, \quad 0 < t < \infty \\
&\text{IC:} \quad u(x,0) = e^{-x^2}, && -\infty \leqslant x \leqslant \infty
\end{aligned}$$

Check your answer.

Problem 5.3 (Transmission-Line Equation - 6 Points)

Derive the transmission-line equation

$$\nu_{xx} = CL\nu_{tt} + (CR + GL)\nu_t + GR\nu$$

for ν from the system of two first-order equations

$$\begin{aligned}
i_x + C\nu_t + G\nu &= 0 \\
\nu_x + Li_t + Ri &= 0.
\end{aligned}$$

Here C, L, R and G are given constants.

Problem 5.4 (Wave Equation - 5 Points)

How many solutions of $u_{tt} = u_{xx}$ can you find by looking for solutions of the form

$$u(x,t) = e^{ax+bt}$$

Is the sum of two solutions a solution?

Deadline for submission: Wednesday, January 11, 10:15 am