



PDE and Boundary-Value Problems (Winter Term 2016/2017)
Assignment H7 - Homework

Problem 7.1 (Wave Problem in Polar Coordinates - 8+6=14 Points)

a) Solve the wave problem in polar coordinates

$$\text{PDE: } u_{tt} = \Delta u, \quad 0 < r < 1, \quad 0 \leq \theta < 2\pi, \quad 0 < t < \infty$$

$$\text{BC: } u(1, \theta, t) = 0, \quad 0 < t < \infty$$

$$\text{ICs: } \begin{cases} u(r, \theta, 0) = J_0(2.4r) - 0.5J_0(8.65r) + 0.25J_0(14.93r), \\ u_t(r, \theta, 0) = 0, \end{cases} \quad 0 \leq r \leq 1$$

by using the method of separation of variables.

b) Sketch the nodal lines for fundamental vibrations U_{34} , U_{33} , and U_{24} .

Problem 7.2 (3-Dimensional Wave Equation - 8 Points)

Solve the problem

$$\begin{aligned} u_{tt} &= c^2(u_{xx} + u_{yy} + u_{zz}), \quad (x, y, z) \in \mathbb{R}^3, \quad t > 0, \\ u(x, y, z, 0) &= 0 \\ u_t(x, y, z, 0) &= \psi(x, y, z) \end{aligned}$$

by the Fourier transform.

Problem 7.3 (2-Dimensional Wave Equation - 5 Points)

Let $u(x, y, t)$ be the solution of the two-dimensional wave equation

$$u_{tt} = c^2(u_{xx} + u_{yy}), \quad (x, y) \in \mathbb{R}^2, \quad 0 < t, \infty$$

with initial conditions

$$u(x, y, 0) \equiv 0$$
$$u_t(x, y, 0) = \begin{cases} 1, & \text{if } x^2 + y^2 \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find $u(0, 0, t)$ for all $t > 0$ and determine the behavior of $u(0, 0, t)$ when $t \rightarrow \infty$.

Problem 7.4 (The Laplacian - 4 Points)

Transform the three-dimensional Laplacian into cylindrical coordinates and spherical coordinates, respectively.

Problem 7.5 (The Exterior Dirichlet Problem - 8 Points)

What is the solution to the exterior Dirichlet problem

$$\begin{array}{ll} \text{PDE} & \Delta u = 0, \\ \text{BC} & u(1, \theta, \phi) = 1 + \cos \phi \end{array} \quad 1 < r < \infty$$

Deadline for submission: Wednesday, February 8, 10:15 am