

> with(inttrans) :  
 > with(student, changevar) :  
 > with(plots);  
 [animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra\_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

> alias(u = u(x, t), U = U(k, t))

Point, u, U (1)

> eq := diff(u, t) - diff(u, x, x) = 0

$$eq := \frac{\partial}{\partial t} u - \left( \frac{\partial^2}{\partial x^2} u \right) = 0 \quad (3)$$

> eq1 := subs(fourier(u, x, k) = U, fourier(eq, x, k));

$$eq1 := k^2 U + \frac{\partial}{\partial t} U = 0 \quad (4)$$

> dsolve(eq1, U);

$$U = \_F1(k) e^{-k^2 t} \quad (5)$$

> subs(\_F1(k) = F(k), %)

$$U = F(k) e^{-k^2 t} \quad (6)$$

> Su := u = invfourier(rhs(%), k, x)

$$Su := u = \text{invfourier}(F(k) e^{-k^2 t}, k, x) \quad (7)$$

> convert(Su, int);

$$u = \frac{1}{2} \frac{\int_{-\infty}^{\infty} F(k) e^{-k^2 t + I k x} dk}{\pi} \quad (8)$$

> assume(k > 0); assume(t > 0); Su := 1/(2\*pi) Int(Int(f(xi) \* exp(-k^2 \* t - I \* k \* xi + I \* k \* x), k = -infinity..infinity), xi = -infinity..infinity);

$$Su := \frac{1}{2} \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi) e^{-k^2 t - I k \xi + I k x} dk d\xi}{\pi} \quad (9)$$

> innerint := int(exp(-k^2 \* t - I \* k \* xi + I \* k \* x), k = -infinity..infinity);

$$innerint := \frac{e^{-\frac{1}{4} \frac{(-\xi + x)^2}{t}} \sqrt{\pi}}{\sqrt{t}} \quad (10)$$

>  $Su := simplify\left( Int\left( f(xi) \cdot simplify\left( \frac{innerint}{2 \cdot pi} \right), xi = -infinity .. infinity \right) \right);$

$$Su := \int_{-\infty}^{\infty} \frac{1}{2} \frac{f(\xi) e^{-\frac{1}{4} \frac{(-\xi + x)^2}{t}} \sqrt{\pi}}{\sqrt{t} \pi} d\xi \quad (11)$$

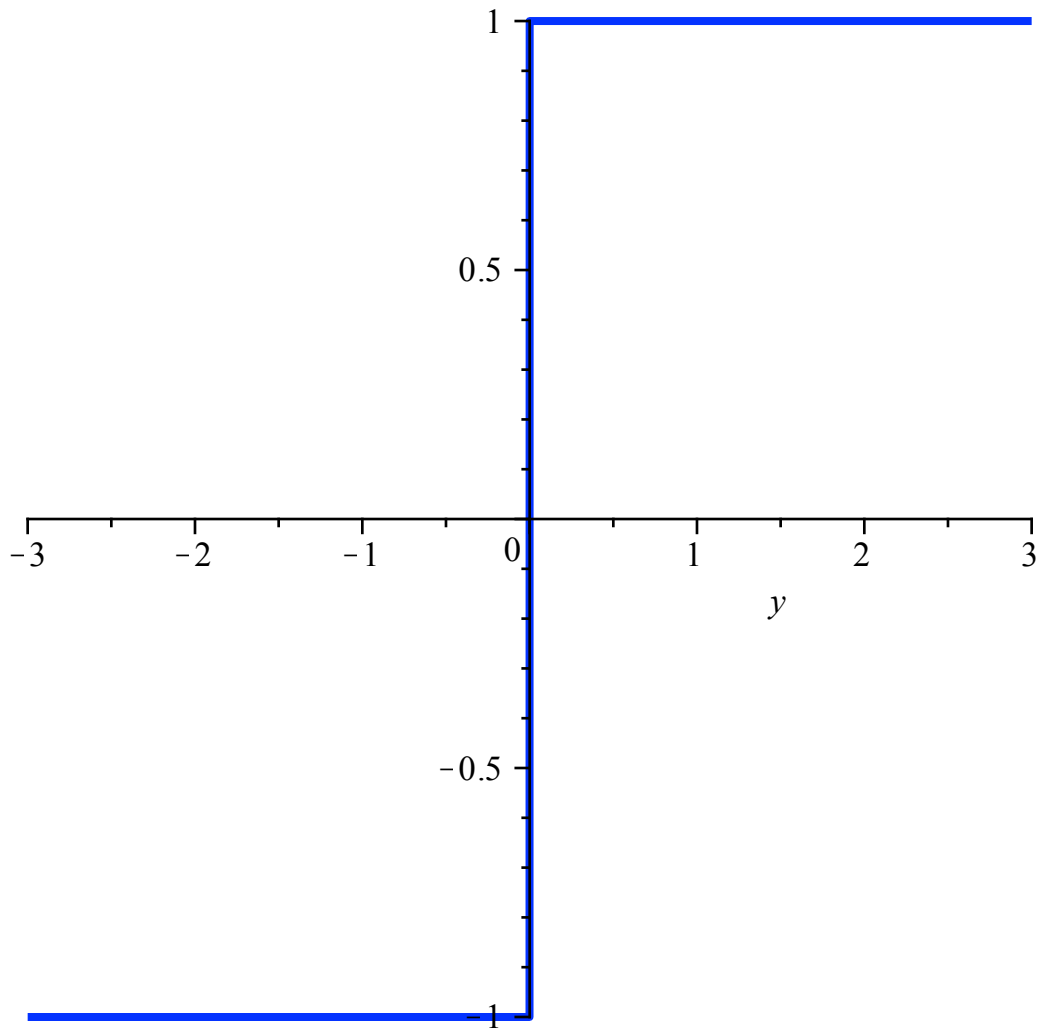
>  $u = \%$

$$u(x, t) = \int_{-\infty}^{\infty} \frac{1}{2} \frac{f(\xi) e^{-\frac{1}{4} \frac{(-\xi + x)^2}{t}} \sqrt{\pi}}{\sqrt{t} \pi} d\xi \quad (12)$$

>  $fl := y \rightarrow Heaviside(y) - Heaviside(-y)$

$fl := y \rightarrow Heaviside(y) - Heaviside(-y) \quad (13)$

>  $plot(fl(y), y = -3 .. 3, thickness = 3, color = blue)$



>  $Su1 = \text{simplify}(\text{value}(\text{subs}(f=f1, Su)));$

$$Su1 = \frac{\text{erf}\left(\frac{1}{2} \frac{x}{\sqrt{t}}\right) \pi}{\pi}$$

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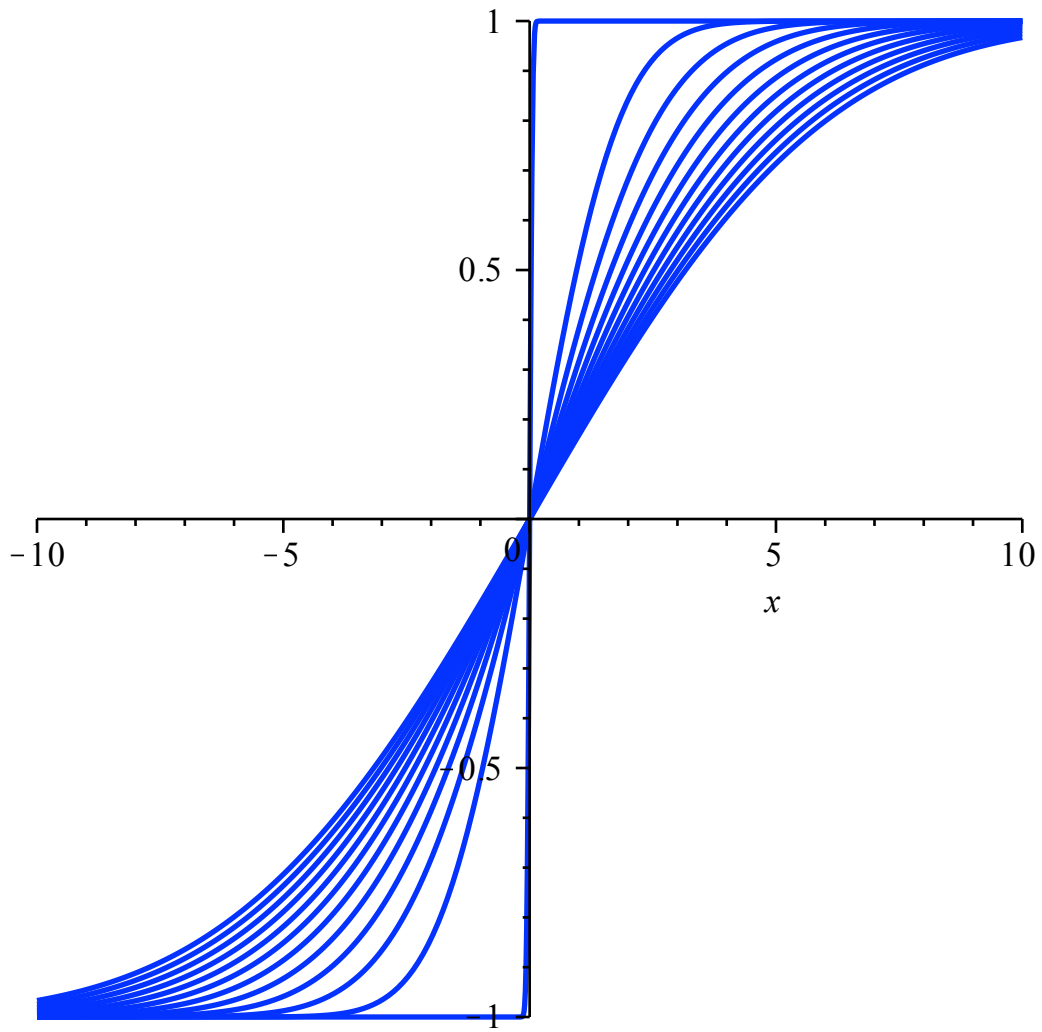
>  $u1 := (x, t) \rightarrow \text{erf}\left(\frac{1}{2} \frac{x}{\text{sqrt}(t)}\right)$

$$u1 := (x, t) \rightarrow \text{erf}\left(\frac{1}{2} \frac{x}{\sqrt{t}}\right)$$

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>  $p1 := \text{seq}(\text{plot}(u1(x, i), x = -10..10, \text{color} = \text{blue}, \text{thickness} = 2), i = 0.001..12) :$

>  $\text{display}([p1])$

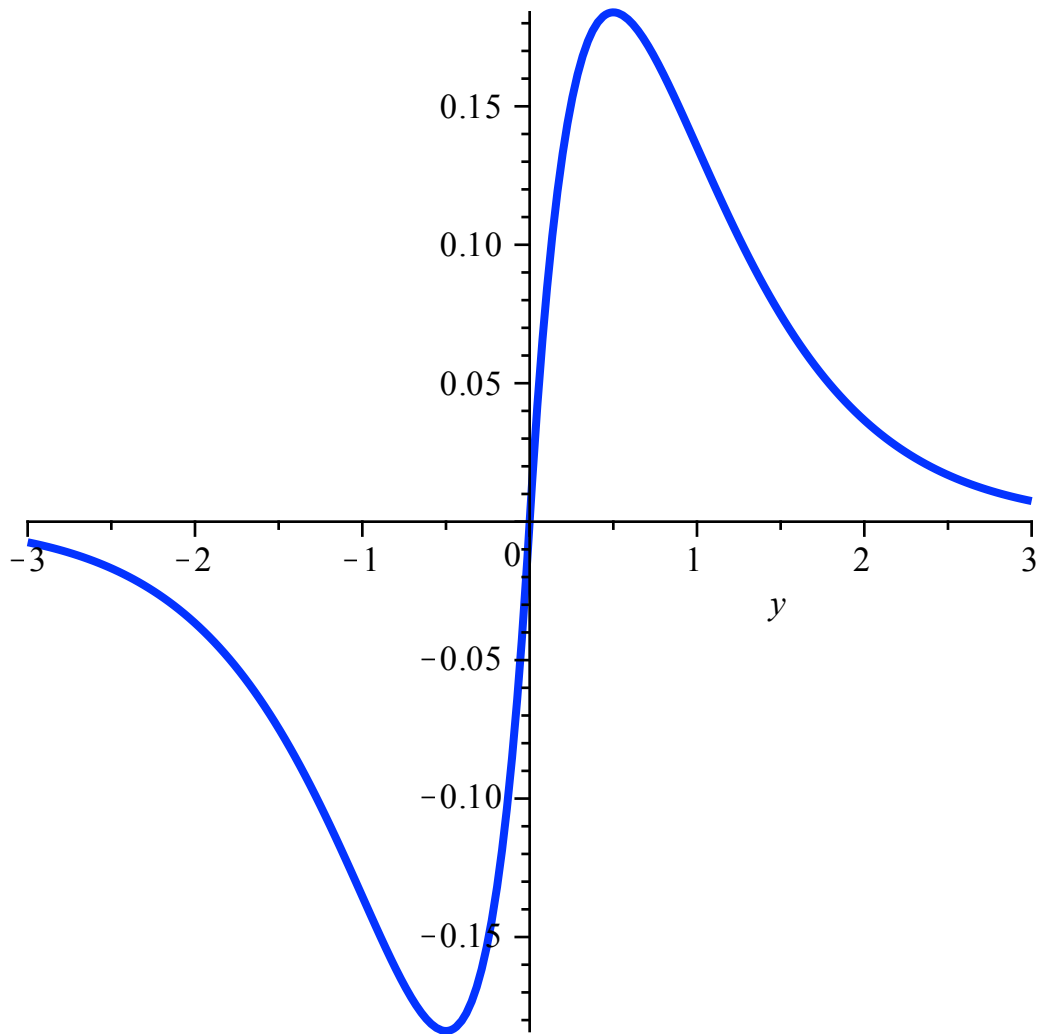


```
> f2 := y -> y · exp(-2 · abs(y))
```

```
f2 := y -> y e-2 |y|
```

```
> plot(f2(y), y = -3 .. 3, thickness = 3, color = blue)
```

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> Su2 = simplify(value(subs(f=f2, Su)));

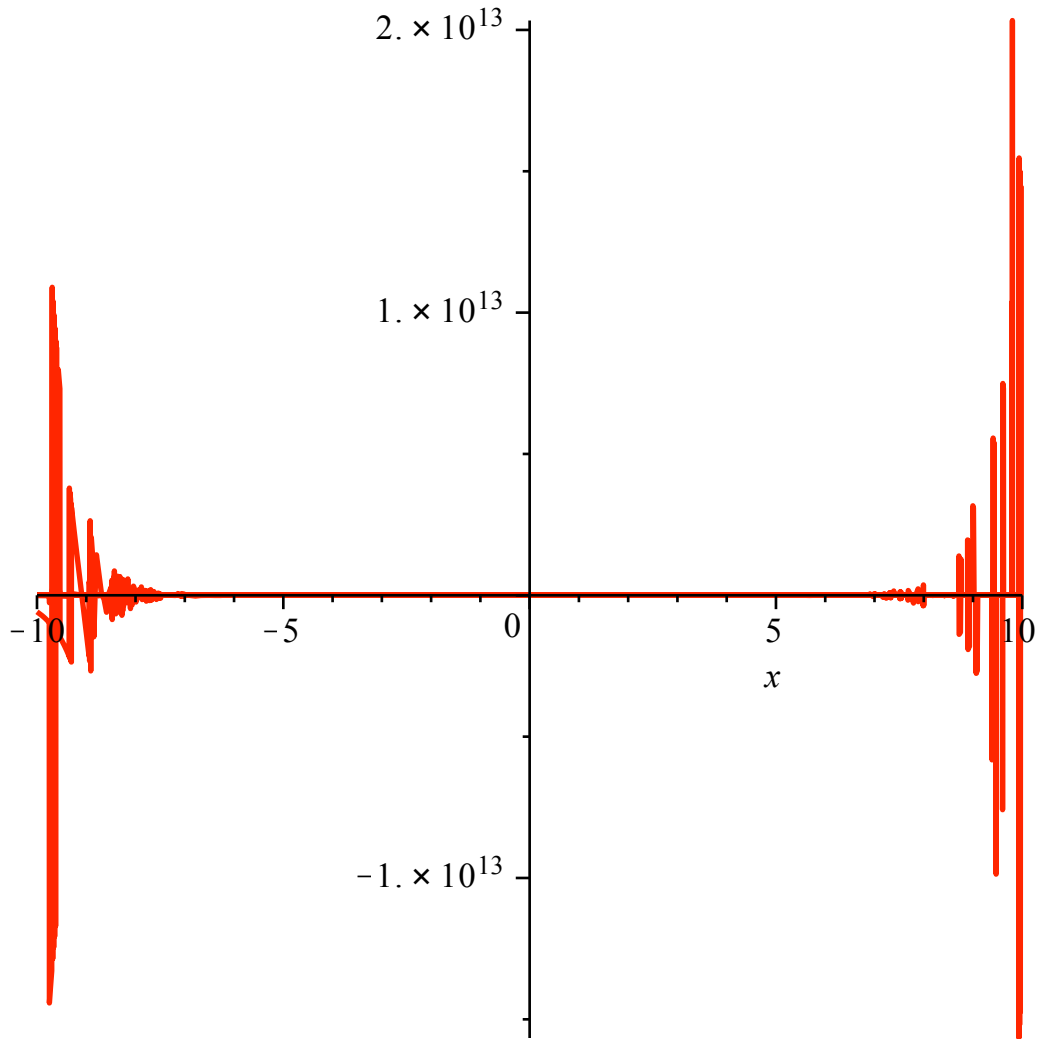
$$Su2 = \frac{1}{2} \frac{1}{\pi} \left( \pi e^{-2x+4t} \left( e^{4x} x + 4 e^{4x} t - \operatorname{erf}\left(\frac{1}{2} \frac{x+4t}{\sqrt{t}}\right) e^{4x} x + x - 4t - x \operatorname{erf}\left(\frac{1}{2} \frac{-x+4t}{\sqrt{t}}\right) + 4t \operatorname{erf}\left(\frac{1}{2} \frac{-x+4t}{\sqrt{t}}\right) - 4t \operatorname{erf}\left(\frac{1}{2} \frac{x+4t}{\sqrt{t}}\right) e^{4x} \right) \right) \quad (17)$$

> u2 := (x, t) →  $\frac{1}{2 \cdot \pi} \cdot \left( \pi \cdot \exp(-2 \cdot x + 4 \cdot t) \cdot \left( \exp(4 \cdot x) \cdot x + 4 \cdot \exp(4 \cdot x) \cdot t - \operatorname{erf}\left(\frac{x + 4 \cdot t}{2 \cdot \sqrt{t}}\right) \cdot \exp(4 \cdot x) \cdot x + x - 4 \cdot t - x \cdot \operatorname{erf}\left(\frac{-x + 4 \cdot t}{2 \cdot \sqrt{t}}\right) + 4 \cdot t \cdot \operatorname{erf}\left(\frac{-x + 4 \cdot t}{2 \cdot \sqrt{t}}\right) - 4 \cdot t \cdot \operatorname{erf}\left(\frac{x + 4 \cdot t}{2 \cdot \sqrt{t}}\right) \cdot \exp(4 \cdot x) \right) \right)$

$$u2 := (x, t) \rightarrow \frac{1}{2} \frac{1}{\pi} \left( \pi e^{-2x+4t} \left( e^{4x} x + 4 e^{4x} t - \operatorname{erf}\left(\frac{1}{2} \frac{x+4t}{\sqrt{t}}\right) e^{4x} x + x - 4t - x \operatorname{erf}\left(\frac{1}{2} \frac{-x+4t}{\sqrt{t}}\right) + 4t \operatorname{erf}\left(\frac{1}{2} \frac{-x+4t}{\sqrt{t}}\right) - 4t \operatorname{erf}\left(\frac{1}{2} \frac{x+4t}{\sqrt{t}}\right) e^{4x} \right) \right) \quad (18)$$

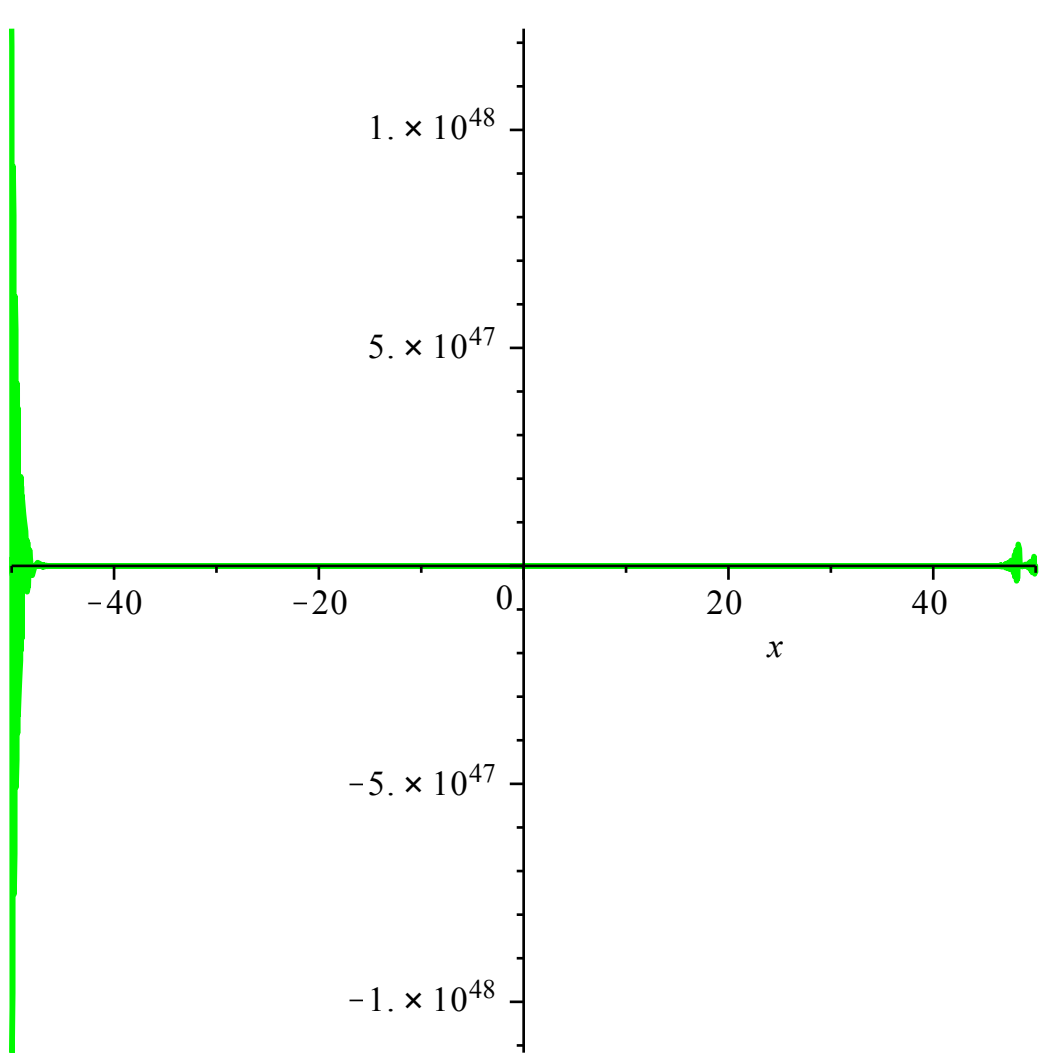
> p2 := seq(plot(u2(x, i), x = -10..10, color = red, thickness = 2), i = 0.001..12) :

```
> display([p2])
```

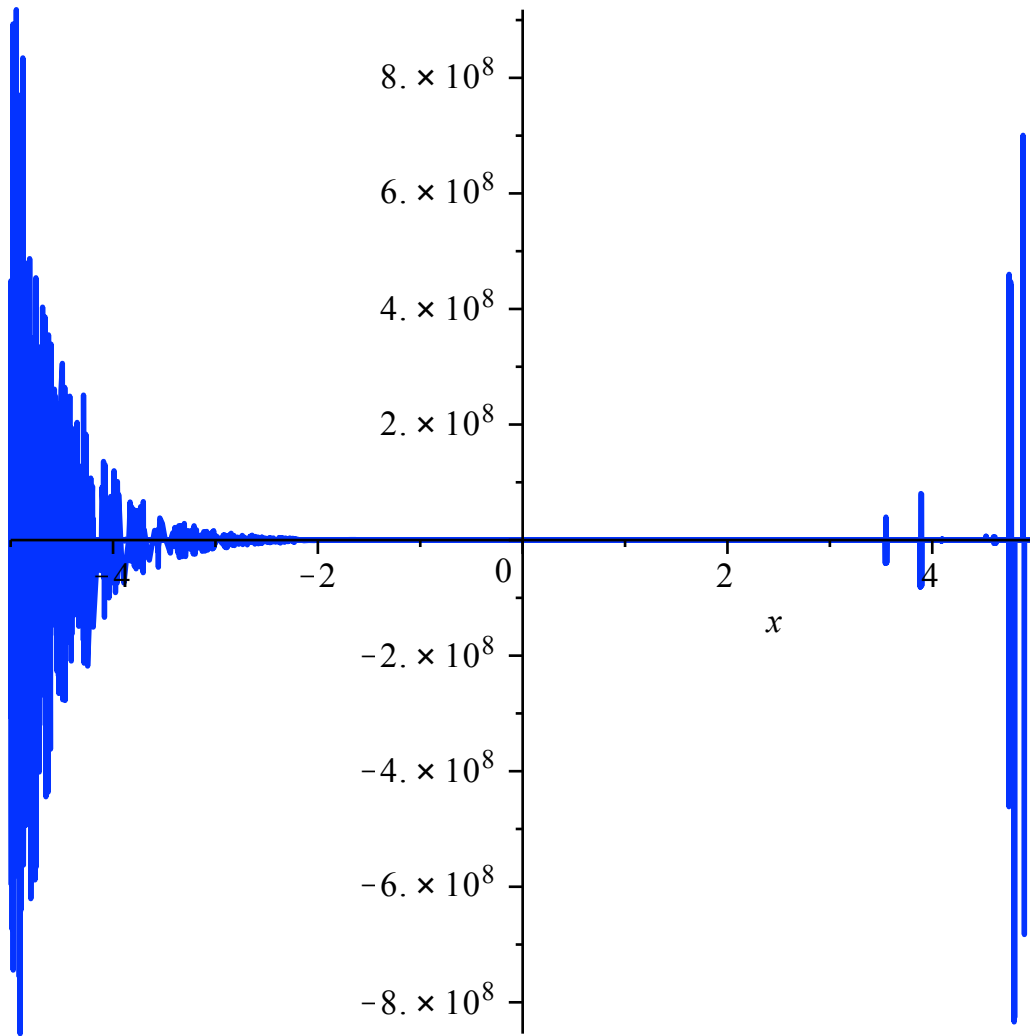


```
> p3 := seq(plot(u2(x, i), x = -50 .. 50, color = green, thickness = 2), i = 0.001 .. 12) :
```

```
> display([p3])
```



```
> p4 := seq(plot(u2(x, i), x=-5..5, color=blue, thickness=2), i=0.001..12) :  
> display([p4])
```



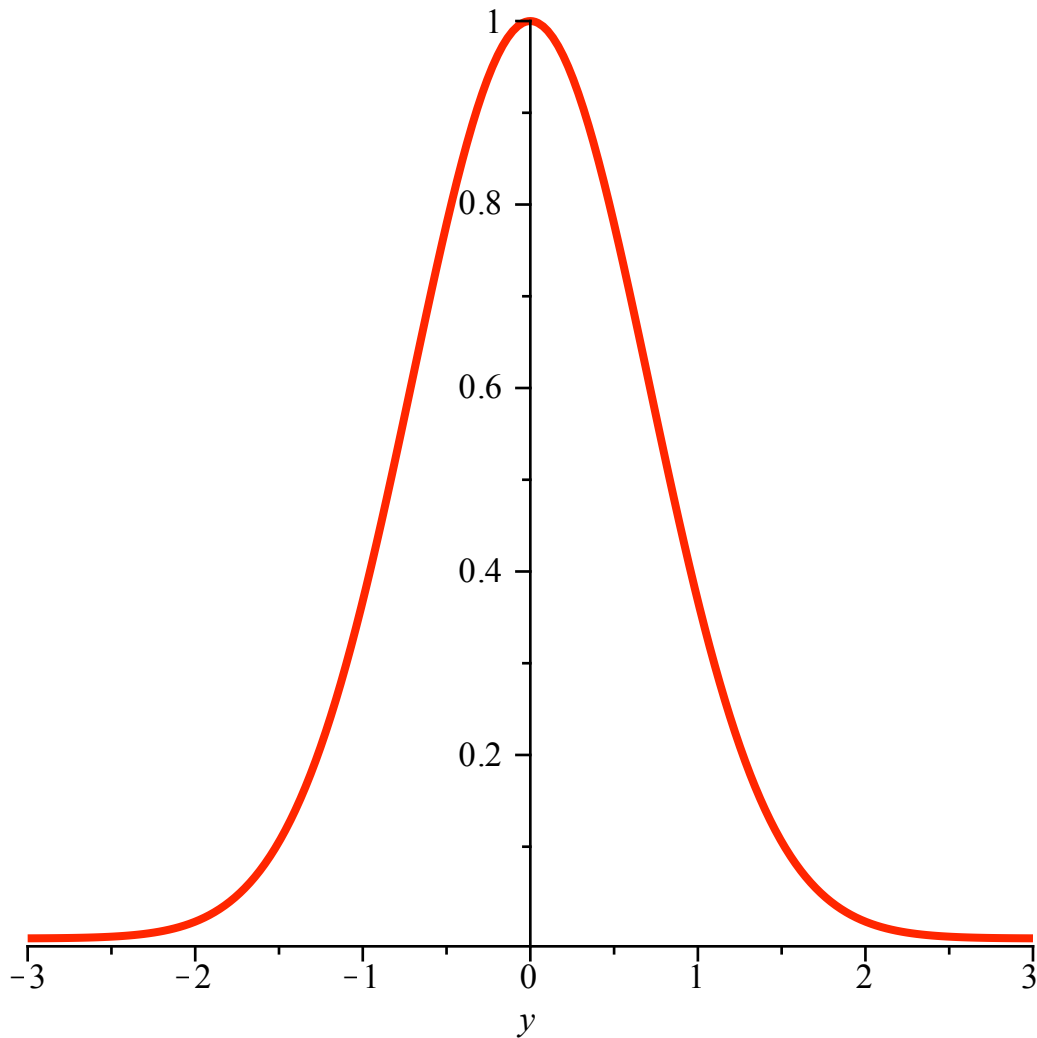
```
> f3 := y -> exp(-y^2);
```

```
f3 := y -> e^-y^2
```

```
> plot(f3(y), y = -3 .. 3, thickness = 3, color = red);
```

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>  $Su3 := \text{simplify}(\text{value}(\text{subs}(f=f3, Su)))$ ;

$$Su3 := \frac{\pi e^{-\frac{x^2}{4t+1}}}{\pi \sqrt{4t+1}}$$

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>  $u3 := (x, t) \rightarrow \frac{\exp\left(-\frac{x^2}{4t+1}\right)}{\text{sqrt}(4t+1)}$

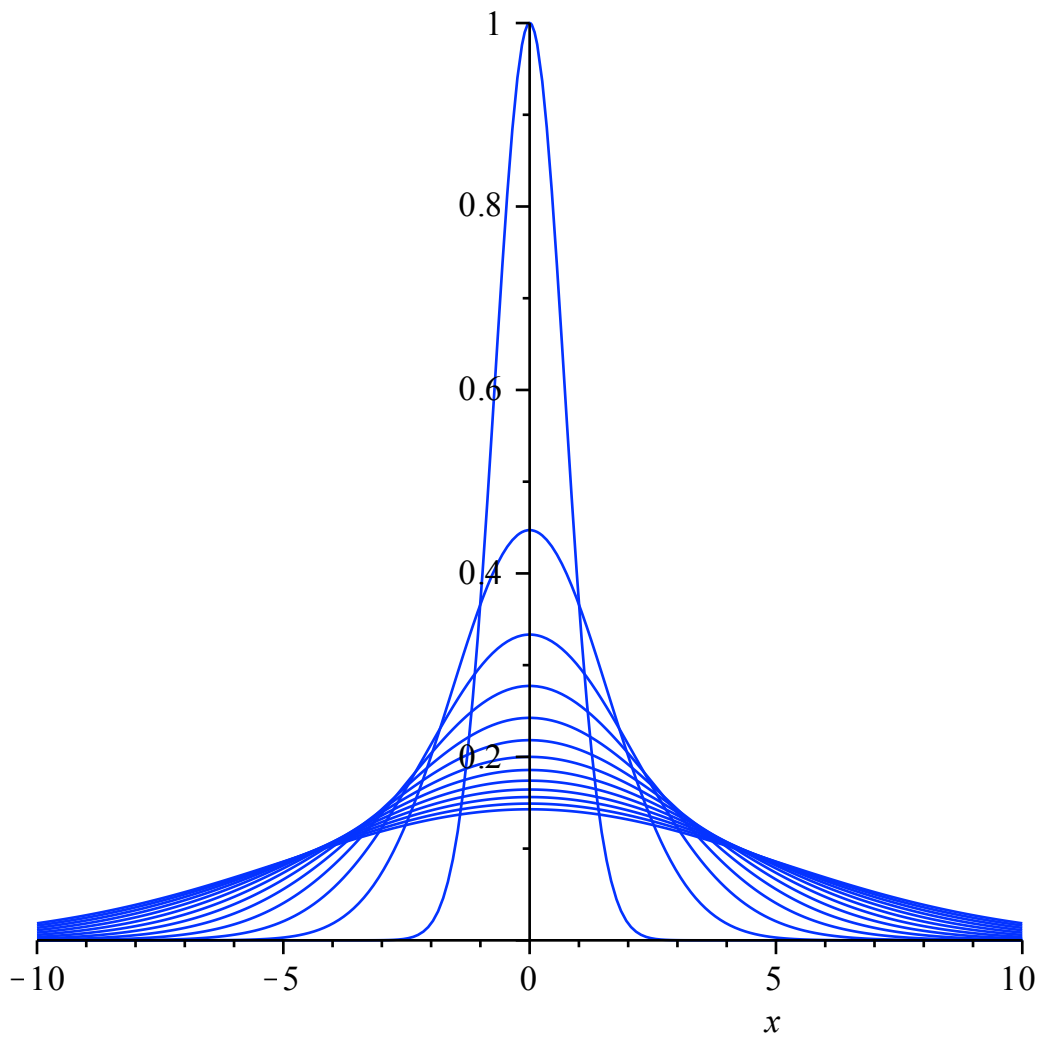
$$u3 := (x, t) \rightarrow \frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}$$

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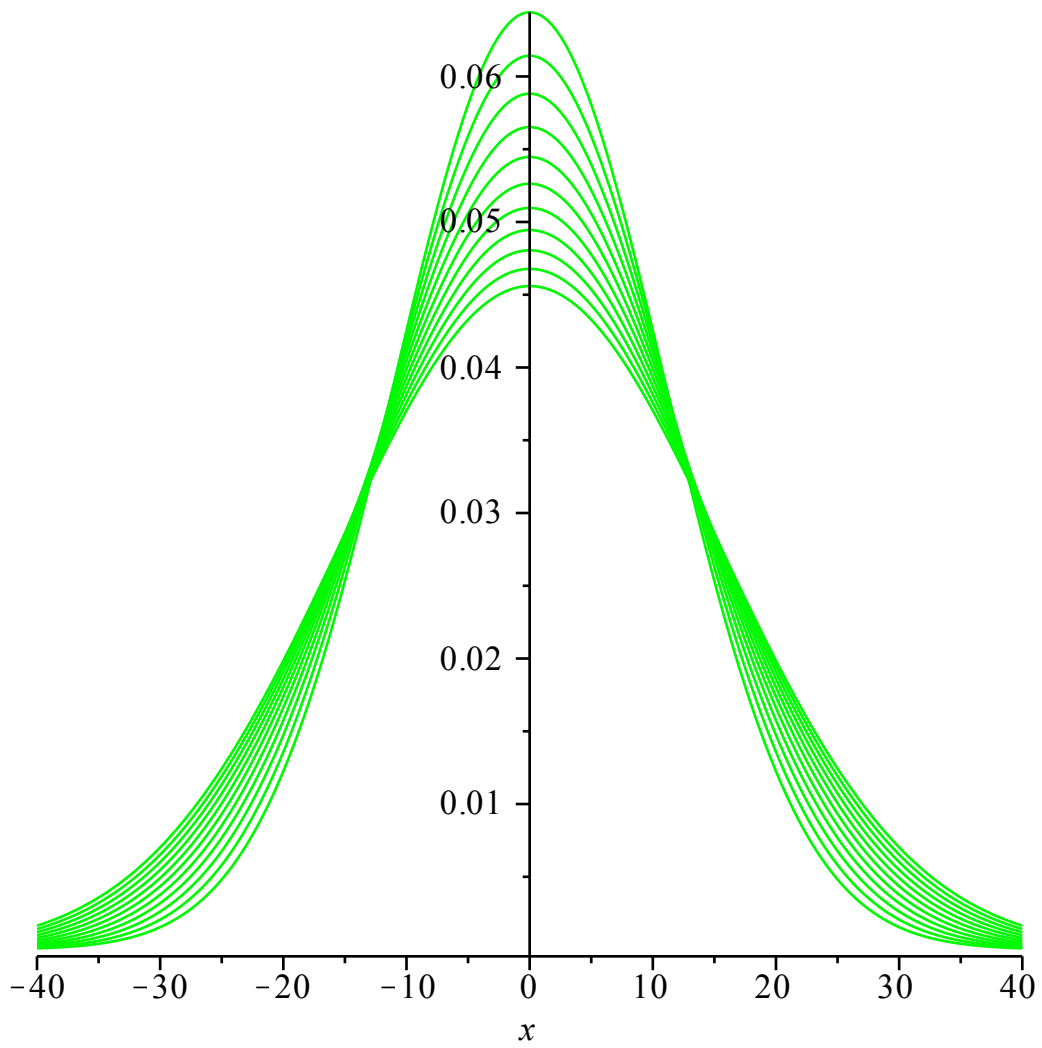
>  $p5 := \text{seq}(\text{plot}(u3(x, i), x=-10..10, \text{color} = \text{blue}, \text{thickness} = 1), i = 0..12)$  :

>  $p6 := \text{seq}(\text{plot}(u3(x, 6\cdot i), x=-40..40, \text{color} = \text{green}, \text{thickness} = 1), i = 10..20)$  :

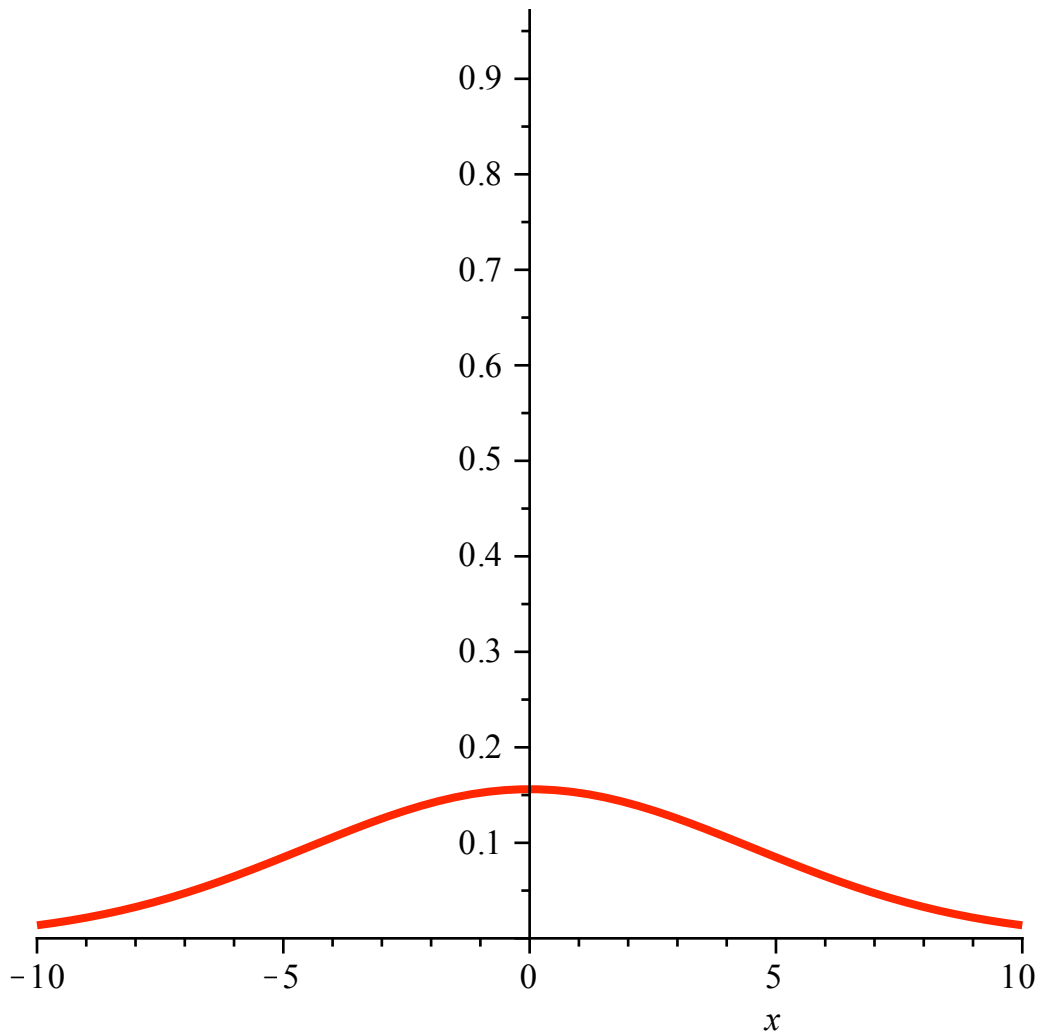
>  $\text{display}([p5])$ ;



```
> display([p6]);
```

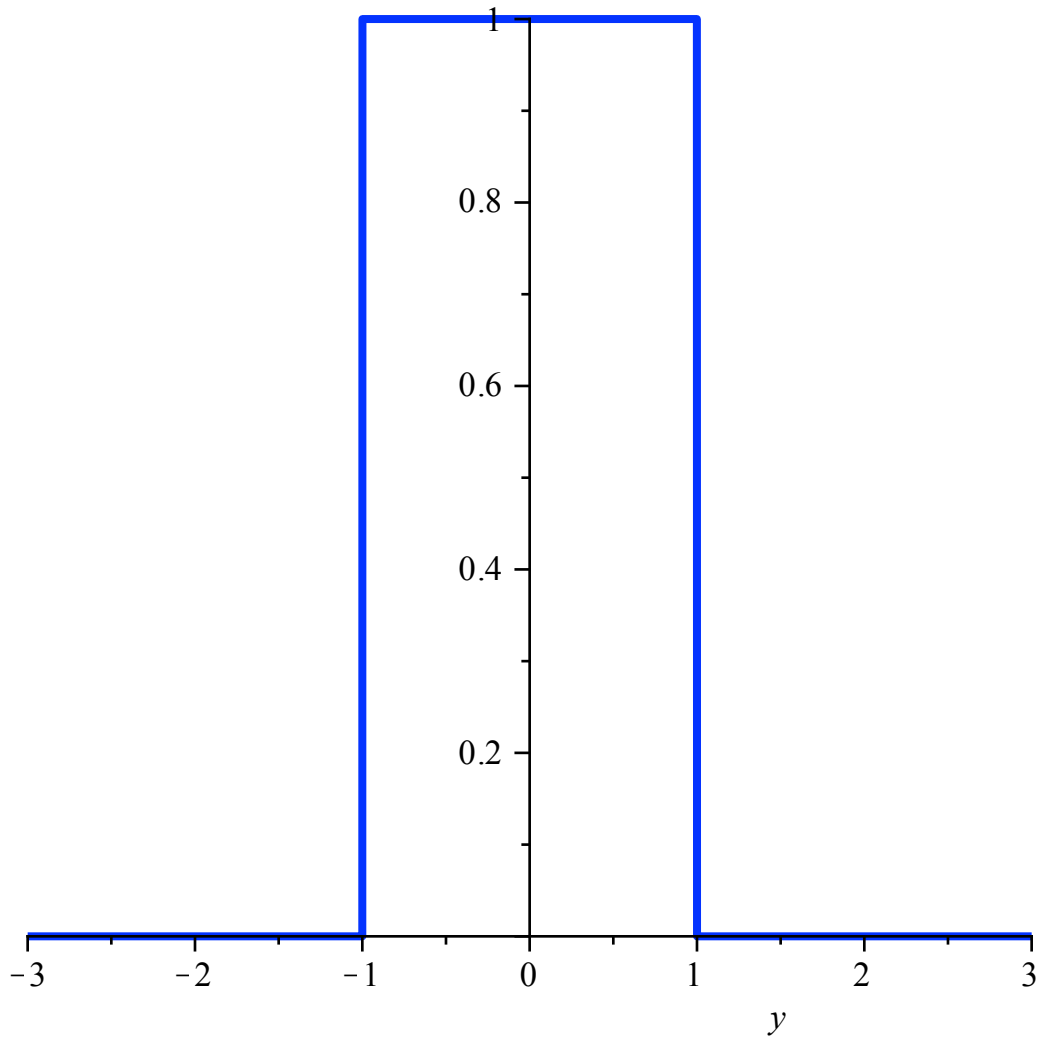


```
> animate(u3(x, i), x = -10..10, i = 0..10, thickness = 3, numpoints = 60)
```



```
> f4 := y→Heaviside(y + 1) - Heaviside(y - 1)  
      f4 := y→Heaviside(y + 1) - Heaviside(y - 1)  
> plot(f4(y), y=-3 ..3, thickness = 3, color = blue)
```

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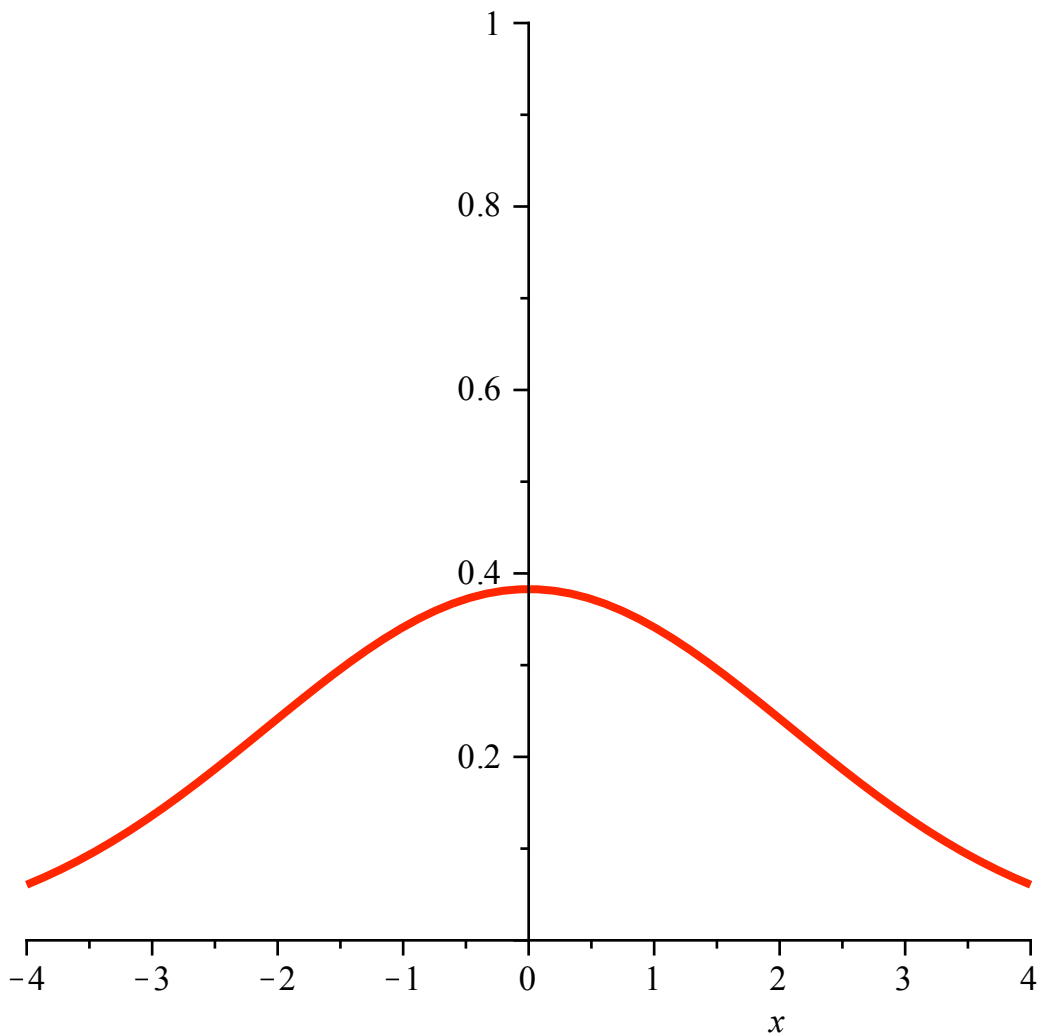
> `Su4 := simplify(value(subs(f=f4, Su)));`

$$Su4 := \frac{1}{2} \frac{\pi \left( -\operatorname{erf}\left(\frac{1}{2} \frac{-1+x}{\sqrt{t}}\right) + \operatorname{erf}\left(\frac{1}{2} \frac{1+x}{\sqrt{t}}\right) \right)}{\pi} \quad (23)$$

> `u4 := (x, t) → 1/2 ( -erf( (x-1)/(2*sqrt(t)) ) + erf( (x+1)/(2*sqrt(t)) ) )`

$$u4 := (x, t) \rightarrow -\frac{1}{2} \operatorname{erf}\left(\frac{1}{2} \frac{x-1}{\sqrt{t}}\right) + \frac{1}{2} \operatorname{erf}\left(\frac{1}{2} \frac{x+1}{\sqrt{t}}\right) \quad (24)$$

> `animate(u4(x, i), x=-4..4, i=0.001..2, color=red, thickness=3, numpoints=60);`



[>

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